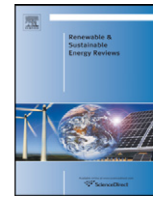
Renewable and Sustainable Energy Reviews 13 (2009) 933–955

Contents lists available at ScienceDirect

Renewable and Sustainable Energy Reviews

journal homepage: www.elsevier.com/locate/rser



A review of wind speed probability distributions used in wind energy analysis Case studies in the Canary Islands

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ARTICLE INFO

Article history:
Received 2 April 2008
Accepted 12 May 2008

Keywords:
Probability density distribution
Wind power density
Coefficient of determination
Moments method
Maximum likelihood method
Least squares method

ABSTRACT

The probability density function (PDF) of wind speed is important in numerous wind energy applications. A large number of studies have been published in scientific literature related to renewable energies that propose the use of a variety of PDFs to describe wind speed frequency distributions. In this paper a review of these PDFs is carried out. The flexibility and usefulness of the PDFs in the description of different wind regimes (high frequencies of null winds, unimodal, bimodal, bitangential regimes, etc.) is analysed for a wide collection of models. Likewise, the methods that have been used to estimate the parameters on which these models depend are reviewed and the degree of complexity of the estimation is analysed in function of the model selected: these are the method of moments (MM), the maximum likelihood method (MLM) and the least squares method (LSM). In addition, a review is conducted of the statistical tests employed to see whether a sample of wind data comes from a population with a particular probability distribution. With the purpose of cataloguing the various PDFs, a comparison is made between them and the two parameter Weibull distribution (W.pdf), which has been the most widely used and accepted distribution in the specialised literature on wind energy and other renewable energy sources. This comparison is based on: (a) an analysis of the degree of fit of the continuous cumulative distribution functions (CDFs) for wind speed to the cumulative relative frequency histograms of hourly mean wind speeds recorded at weather stations located in the Canarian Archipelago; (b) an analysis of the degree of fit of the CDFs for wind power density to the cumulative relative frequency histograms of the cube of hourly mean wind speeds recorded at the aforementioned weather stations.

The suitability of the distributions is judged from the coefficient of determination R^2.

Amongst the various conclusions obtained, it can be stated that the W.pdf presents a series of advantages with respect to the other PDFs analysed. However, the W.pdf cannot represent all the wind regimes encountered in nature such as, for example, those with high percentages of null wind speeds, bimodal distributions, etc. Therefore, its generalised use is not justified and it will be necessary to select the appropriate PDF for each wind regime in order to minimise errors in the estimation of the energy produced by a WECS (wind energy conversion system). In this sense, the extensive collection of PDFs proposed in this paper comprises a valuable catalogue.

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doi:10.1016/j.rser.2008.05.005

1. Introduction

The probability density function (PDF) of wind speed is important in numerous wind energy applications [1,2]. From the 1940s to the present day, a large number of studies have been published in the scientific literature related to renewable energies that propose the use of a variety of PDFs to describe wind speed frequency distributions [1–197,199–205]. These proposals range from standard parametric distributions [1–11,21–61,63–185,187–189,197,199] to distributions generated by applying the principle of maximum entropy [154,204,205]. Both bivariable [12–20,147,202] and univariable distributions [1–11,21–61,63–146,148–197,199–205] have been proposed. The suggestion has also been made to use unimodal [1–153,156–197,199], bimodal and bitangential[1] [20,154,155,200–205] and hybrid distributions [186,190–196].

A review is carried out in this paper of a wide collection of PDFs which have been proposed for wind energy analysis, as well as of other new ones which are considered here. Likewise, the methods are reviewed which have been proposed to estimate the parameters on which these distribution laws depend, as well as the statistical tests used to see whether a sample of wind data comes from a population with a particular probability distribution.

With the aim of cataloguing the goodness of fit of the different models which are included in this paper, a comparison is made between them and the two parameter Weibull distribution, the most widely used and accepted distribution in the specialised literature on wind energy and other renewable energy sources. This comparison is based on: (a) an analysis of the degree of fit of the continuous cumulative distribution functions (CDFs) for wind speed to the cumulative relative frequency histograms of hourly mean wind speeds recorded at weather stations located in the Canarian Archipelago (Spain) [206]; (b) an analysis of the degree of fit of the CDFs for wind speed density[2] to the cumulative relative

[1] This occurs if there are two distinct points, v_1, v_2, at which there is a common tangent to the density curve. Thus, bitangentiality is implied by, but does not imply bimodality. Informally, bimodality implies an extra hump, but bitangentiality merely an extra bump [201].

[2] The PDFs for wind power density allow notice to be given of the importance of the winds of intensity between two given values, from the point of view of power production. The mode of the PDF for wind power density shows the most energetic ranges of wind speed at the design site. Thus, this mode can be used as a wind turbine design parameter, as annual energy production is usually a maximum if a wind turbine is designed for maximum aerodynamic efficiency at this mode [174,182,185,202].

Nomenclature

a,b	interval limits where a PDF is defined
$B(\cdot,\cdot)$	beta function
B.pdf	three parameter beta probability density function dependent on wind speed v and a set of parameters ϕ
CDF	cumulative distribution function
$f(v,\boldsymbol{\phi}),\ f(v)$	non-hybrid probability density function
fr	a vector which contains the relative frequencies corresponding to the N intervals in which the n wind speed data of a sample are classified. Its components are expressed: fr_i ($i = 1, \ldots, n$)
$F(v,\boldsymbol{\phi}), F(v)$	cumulative distribution function dependent on wind speed v and a set of parameters ϕ.
G.pdf	two parameter gamma probability density function
GG.pdf	three parameter generalised gamma probability density function
$h(v,\boldsymbol{\phi},\theta_0), h(v)$	hybrid probability density function dependent on wind speed v and a set of parameters ϕ and θ_0. Eq. (2)
$H(v,\boldsymbol{\phi},\theta_0), H(v)$	hybrid cumulative distribution function dependent on wind speed v and a set of parameters ϕ and θ_0. Eq. (4)
IG.pdf	two parameter inverse Gaussian probability density function
$I_0(\alpha,\beta)$	function defined by Eq. (39)
Kt	Coefficient of Kurtosis of a sample of wind speeds
$L(\mathbf{V}, \phi)$	likelihood function of a PDF
LN.pdf	two parameter lognormal probability density function
LSM	least squares method
$m, m'_2, m'_3, \ldots$	low-order statistical moments with respect to the origin of a sample of n wind speeds
$m_2, m_3, \ldots$	low-order statistical moments with respect to the mean m of a sample of n wind speeds. Also low-order statistical moments with respect to the mean m of a sample of napierian logarithms of wind speeds. Eq. (8), (9) and (11)
M	number of low-order statistical moments with respect to the origin of a sample of wind speeds
Me	median of a sample of wind speeds
MEP	maximum entropy principle
MLM	maximum likelihood method
MM	moment method
MTNW.pdf	mixture of NT.pdf and W.pdf probability density function
MWW.pdf	two components mixture Weibull probability density function
$\mathrm{ME_M}$.pdf	$M+1$ parameter maximum entropy probability density function
n	number of wind speed data of a sample
N	number of intervals in which the n observed wind speed values of a sample are grouped
NT.pdf	two parameter singly truncated from below normal probability density function
P	a vector which contains the experimental cumulative relative frequencies of wind speeds. Its components are expressed: P_i ($i = 1, \ldots, n$) Eq. (1)
PDF	Probability distribution function.
R.pdf	one parameter Rayleigh probability density function
R^2_{wp}	coefficient of determination which indicates the degree of fit between a CDF for wind speed and the cumulative histogram of relative frequency of the cube of hourly mean wind speeds recorded at a given weather station
R^2_{ws}	coefficient of determination which indicates the degree of fit between a CDF for wind speed and the cumulative histogram of relative frequency of hourly mean wind speeds recorded at a given weather station
$R^2_{\mathrm{wp,h}}$	coefficient of determination which indicates the degree of fit between a hybrid CDF for wind speed and the cumulative histogram of relative frequency of the cube of hourly mean wind speeds recorded at a given weather station
$R^2_{\mathrm{ws,h}}$	coefficient of determination which indicates the degree of fit between a hybrid CDF for wind speed and the cumulative histogram of relative frequency of hourly mean wind speeds recorded at a given weather station
s^2	variance of a sample of n wind speed data
$S(V_{\max};\phi)$	non-linear objective function which is to be minimised, Eq. (1)
Sk	coefficient of skewness of a sample of wind speeds
SRN.pdf	two parameter square-root normal probability density function
T_1, T_2 and T_3	Eq. (13)
$v_{\max}$	maximum recorded wind speed of a sample
$v_{\min}$	minimum recorded wind speed of a sample
V	vector which contains the n wind speed data of a sample. Its components are expressed: $v_i (i = 1, \ldots, n)$
$\mathbf{V}_{\max}$	a vector which contains the maximum recorded wind speed values within the N intervals in which the n wind speed data of a sample are classified. Its components are expressed: $v_{\max,i} (i = 1, \ldots, n)$ Eq. (1)
W.pdf	two parameter Weibull probability distribution function
WECS	wind energy conversion system
$\mathrm{Wp}(v,\boldsymbol{\phi},\theta_0)$	PDF of the wind power density. Eq. (5)
X	vector whose components are the napierian logarithms of the components of the vector $\mathbf{V}_{\max}$. Eq. (27)
Y	vector which contains data related to the vector **P**. Eq. (27)
$Z(\cdot)$	function defined by Eq. (44)

Greek letters

α	parameter of the PDFs: GG.pdf, W.pdf, SRN.pdf, NT.pdf, LN.pdf, IG.pdf, B.pdf, and MNTW.pdf
α_1, β_1	parameters of one of the two W.pdf which make up the MWW.pdf
α_2, β_2	parameters of one of the two W.pdf which make up the MWW.pdf
β	parameter of the PDFs: GG.pdf, G.pdf, W.pdf, SRN.pdf, NT.pdf, LN.pdf, IG.pdf, B.pdf, and MNTW.pdf
$\Gamma(\cdot)$	gamma function
$\delta(v)$	dirac delta. Eq. (3)
ζ	relation of the two parameters of the NT.pdf. Eq. (43)
η	parameters of the PDFs: GG.pdf and G.pdf
θ	parameter of the R.pdf
θ_0	relative frequency of calms (null wind speeds)
λ	vector which contains the parameters of the ME_M.pdf. The components of this vector are: $\lambda_0, \lambda_1, \ldots, \lambda_M$
μ,σ	parameters of the SRN.pdf
ξ	position parameter of the B.pdf
ϕ	a vector which contains the parameters of a probability density function. Eq. (1)
$1-\phi(\cdot)$	function defined by Eq. (44)
$\psi(\cdot)$	digamma function
$\psi'(\cdot)$	trigamma function
ψ''	tetragamma function
ω	mixture parameter of the mixture distributions MWW.pdf and MNTW.pdf. $(0 \le \omega \le 1)$

frequency histograms of the cube of hourly mean wind speeds recorded at the aforementioned weather stations.

The selected stations present different wind speed frequency histograms, allowing an analysis of the flexibility and usefulness of the collection of models in the description of different wind regimes (high frequencies of null wind speeds, unimodal, bimodal regimes, etc.). The suitability of the distributions is judged from the R^2 statistic (or coefficient of determination) used in various studies [20,42,43,50,47,92,149,153–155,189,201,204,205].

The two parameters of the Weibull (W.pdf) are practically the only ones which, in the literature related to renewable energies, have been estimated making use of different estimation methods [1,2,7,8,11,66,67,70,71,91,114,156–158,162,164,194]. In this paper, however, the three most frequently used estimation methods, namely the moments method (MM), the maximum likelihood method (MLM) and the least squares method (LSM), are applied to the selected collection of PDFs. The aim is to analyse the complexity of the estimation and the goodness of fit to the experimental data in function of the chosen estimation method.

2. Overview of proposed wind speed probability density distributions

In the period from 1940 to 1945, special wind-research programs were carried out in the United States with the aim of investigating potential wind-turbine sites [3]. According to Putnam [3], the experience obtained in the construction of wind speed frequency curves had shown that they were all of a similar type and defined by statistics like Pearson Type III functions. After Putnam [3], who was the first to prospect for sites for large wind turbine generators [2], suggested this probabilistic model, a number of significant models have been proposed to represent wind speed probability density functions. In 1951, Sherlock [4] proposed, like Putnam [3], the use of the Pearson Type III curve (gamma distribution function) of two parameters (scale parameter and shape parameter) arguing that: (a) it had been widely and successfully used in many statistical studies of natural phenomena; (b) graphical studies showed that it gave a reasonably good fit to velocity data; (c) tables were readily available for its use. However, this probability distribution has not been extensively used in the field of wind energy, as the paucity of references testifies to [2,5–11].

For some authors who published their research between 1950 and 1970, and from a viewpoint of statistical theory, it is advantageous to describe the horizontal wind in terms of a bivariate distribution system [2]. Essenwanger [12] showed that the elliptical bivariate distribution of two components was useful for the calculation of upper-air wind speed probabilities. Crutcher and Baer [13] studied the special case of the normal distribution of two variables. These two variables were the horizontal components of the wind speed according to a cartesian axes system. This bivariate distribution depends on the means of the wind speed and the standard deviations in accordance with these axes, and on the normalized correlation coefficient that exists between both variables.

The energy crises of the 1970s, caused by the increase in petrol prices, reactivated interest in renewable energies. This is reflected in the increase in the number of probability models which are proposed during this period. Several authors proposed the use of the bivariate normal distribution with wind speed and direction as variables [14–19]. The distribution law of one variable, derived from this normal law of two variables, comprises a sum of Bessel function products [14]. According to Justus [1] and Koeppl [2], this model is difficult to use for wind energy application given its complicated form and its requirement of five parameters. The isotropic Gaussian model of McWilliams et al. [15,16] and Weber [17] was derived from the assumptions that the wind speed component along the prevailing wind direction (longitudinal component of the wind vector) is normally distributed with non-zero mean and a given variance, while the wind speed component along a direction orthogonal to that (lateral component of wind vector) is independent and normally distributed with zero mean and the same variance. The marginal PDF of the wind speed is obtained after integration over the direction variable [20]. The anisotropic Gaussian model of Weber [18–19] is a generalization of the model of McWilliams et al. [15]. In Weber [18], no restrictions are imposed on the standard deviations of the longitudinal and lateral fluctuations. The marginal PDF of the wind speed is obtained after integration over the direction variable [19,20]. The work carried out in the Canary Archipelago by Carta et al. [20] indicate that the marginal PDFs of wind power density derived from isotropic Gaussian and anisotropic Gaussian models provide, in all cases analysed, low coefficient of determination values.

Winger [21,22] proposed a square-root normal model. This model was based on the hypothesis that the square-root transformation of observed wind speeds can be fitted by a Gaussian distribution centred on the mean of the square-root of the observed speeds. This distribution has had little repercussion on later wind energy analyses [11,23].

Also from the 1970s came the first proposals for the use of wind speed probability distributions of Rayleigh (R.pdf) and Weibull.

The Rayleigh distribution function of one parameter corresponds to the chi distribution for two degrees of freedom. This distribution coincides with the two parameter Weibull distribution when the shape parameter of the latter takes the value 2. The Rayleigh distribution has been widely used, either exclusively [24–27] or sharing centre stage with the Weibull distribution [2,7,8,23,28–60] in numerous wind power studies, including regulatory works [61], due to its simplicity. The W.pdf which had occasionally been used in calculations of the loads exerted by winds on structures [62] began to be used in wind power studies at the end of the 1970s [1,7,28–30,63–68]. The characteristics of the two parameter Weibull distribution have given rise to it becoming the most widely used and accepted probability distribution in the specialised literature on wind energy and other renewable energy sources [8,11,23,31–60,69–185]. Indeed: (a) it is included in regulations concerning wind energy [61]; (b) it forms part of the most popular computer modelling packages, such as HOMER [165–167] and WASP [168–172]; (c) it is practically the only distribution law which is recommended in the present day in books related to wind energy [1,2,60,172–185].

The W.pdf has been employed in various regions of the world fundamentally in the evaluation of wind energy potential; or to be more precise, in the statistical analyses of wind characteristics and wind power density [30–47,69–118]. This distribution has also been used in the construction of a variety of models: (a) estimation of the energy output and capacity factor of wind turbines [119–131], (b) estimation of the performance of autonomous wind energy systems [132–136], (c) variation of wind characteristics with height [137–139], (d) wind speed time series characterisation and stochastic simulation and forecasting of wind speed sequences [138,140–146], (e) bivariate analysis of wind speed and direction [147], (f) wind turbine breakdown analysis [148], etc. A number of questions have been raised that need to be taken into account with the use of the models [149–152]. In order to analyse the behaviour of the W.pdf, comparisons have been performed with certain distribution laws [153–155], and different estimation methods of the parameters on which this distribution depends have been proposed [1,2,156–164].

According to Justus [1] and Koeppl [2], from a practical point of view the two parameter Weibull function is more convenient to use than the univariate Gaussian function and more flexible than the Rayleigh function. It is particularly useful for describing the occurrence of high wind speeds [12]. According to Tuller and Brett [186], the selection of the Weibull distribution can often be attributed to its flexibility, providing a good fit to the observed wind speed distributions, and to the fact that only two parameters are needed for estimation.

The lognormal distribution of two parameters, which in the 1970s had been used by Luna and Church [187] in studies on air pollution and by Kaminsky [7] and Justus et al. [65] in wind speed analyses, has only very exceptionally been proposed for use since then [11,153,188].

Lavagnini et al. [6] used, amongst others, the Pearson Type I function (beta distribution) of three parameters in analyses of wind energy using the wind speed values from 48 meteorological stations of Italian Air Force. Carta et al. [189] used the beta distribution, together with 13 other distributions with the aim of analysing the influence of the degree of fit of a density probability function to wind speed data on the WECS (wind energy conversion system) mean power output estimation.

Due to the fact that the W.pdf has been unable to represent all the wind regimes encountered in nature, researchers have continued to propose new wind speed probability models. These have fundamentally consisted of standard parametric distributions. Takle and Brown [190] proposed the use of what is termed a hybrid density probability function for describing wind speed distributions having non-zero probability of null wind speeds (calms). The method used simply removes those measurements determined to be "calm", and fits the Weibull distribution to the non-zero wind speeds. The zero wind speeds are then reintroduced to give the proper mean and variance and renormalize the distribution. This hybrid distribution has been analysed and employed in various studies [10,186,190–196]. However, Tuller and Bret [186] indicated that in the studies that they carried out in seven stations located on the British Columbia coast, they did not observe in general that the hybrid distribution offered much advantage over the ordinary Weibull distribution, and in many cases produced a worse fit to the observed data.

Stewart and Essenwanger [197] proposed a three parameter variant of the Weibull distribution, where the new parameter is called the location parameter. These authors found the three parameter Weibull distribution to be generally superior to the ordinary two parameter Weibull distribution for estimating certain probability thresholds. However, as pointed out by Chade and Sharma [8], the added location parameter introduces difficulties in the estimation and a positive value for it gives rise to an unrealistic condition of zero probability of wind speeds less than the parameter value.

Auwera et al. [10] proposed the use of the three parameter generalised gamma distribution (GG.pdf) (two shape parameters and one scale parameter) for estimation of mean wind power densities. This distribution is a more general case of the two parameter Weibull distribution [198]. According to these authors, the GG.pdf gave a better fit to observed wind speeds than did a number of other distribution functions.

The inverse Gaussian distribution (IG.pdf)[3] is suggested by Bardsley [199] as a useful alternative to the three parameter Weibull distribution with a positive location parameter for the description of wind speed data with low frequencies of low speeds. According to Bardsley [199], a comparison of the two distributions indicates a region of strong similarity. However, the parameters of the IG.pdf are more difficult to calculate when the maximum likelihood method is used. In addition, according to Bardsley [199], the IG.pdf possesses a number of useful features with respect to wind energy evaluation.

A singly truncated from below normal distribution (NT.pdf) has been used by Carta et al. [189] to describe wind regimes with high probabilities of null wind speeds.

In order to describe wind regimes that present bimodality, various authors have proposed the use of two component mixture distributions. The mixture distributions proposed have comprised mixtures of two W.pdfs [155,175,200–202], mixtures of two normal distributions [203] and mixtures of W.pdf and NT.pdf [20,155]. This latter one, in addition to representing bimodal wind regimes, takes into account null wind speeds.

The use has also been proposed, in the estimation of wind energy, of a general probability distribution obtained through application of the maximum entropy principle (MEP), widely used in various fields of science and engineering, constrained by the low-order statistical moments of a given set of observed hourly mean wind speed data [154,204,205]. These PDFs, besides being able to represent unimodal, bimodal and bitangential winds, take into account the frequencies of null wind speeds [154].

[3] Also called the Wald distribution.

3. Methods used for the estimation of parameters and statistical tests

3.1. Methods used in the estimation of parameters

In order to estimate the parameters[4] of the distribution laws used to describe the characteristics of the wind, various procedures have been adopted.

In most of the studies presented in the specialised literature on wind energy and other renewable energy sources, the parameters are considered from the classical point of view[5]; whilst papers which use a Bayesian approach are few [160].[6] It should also be pointed out that in practically all the studies published in the field of renewable energies, point-based rather than interval-based estimation of the parameters has been carried out [151]. In other words, a search is made for an estimator which, based on the sample data, gives rise to a single-valued estimation of the value of the parameters.

There exist different methods which can be used to find point estimators of parameters with desirable properties [206]. However, the moments method [1–5,7,8,11,30,34,36,38,46,66,67,72,-74,76,91,97,113,114,131,139,154,157,161,164,194,195,200,201,2-03–205], the maximum likelihood method [2,7,8,10,66,70,71,-77,79,91,96,101,105,119,151,153,156,158,162,163,164,189–191,-199,201] and the least squares method [1,2,6,11,20,31,–33,37,39,43,59,47,65–67,70,71,73,75,80–83,85,86,91,92,103,114,–117,150,155–158,161,162,164,186,190,192,194,196,197,201,202] are the ones which have normally been employed in the estimations of the distributions used in statistical analyses of wind.

The moments method consists of equating a certain number of statistical moments of the sample with the corresponding population moments [207]. In this way, a number of equalities are obtained which allow determination of the unknown parameters of the distribution. The estimators obtained through the method of moments are consistent, but not centred, nor with minimum variance nor robust. The advantage of these estimators lies in their simplicity. The drawback is that this method does not use all the information from the sample. According to Canavos [207], the moments method provides a reasonable alternative when estimators of maximum likelihood cannot be determined.

In essence, the maximum likelihood method of estimation selects as estimators those values of the parameters which have the property of maximising the value of the probability of the randomly observed sample [207]. In other words, the maximum likelihood method consists of finding the values of the parameters which maximise the likelihood function. This method provides estimators which are asymptotically centred, have normal asymptotic distribution, and are efficient (asymptotically of minimum variance).

The least squares method provides an alternative to the maximum likelihood method, though it has less optimum desirable properties than the latter [207]. In least squares estimation, the unknown values of the parameters of a probability density function can be estimated by looking for the numerical values of the parameters which minimise the sum of the squares of the deviations between the experimental values and those obtained with the model. Normally, the LSM is applied to the cumulative distribution function $F(v, \boldsymbol{\phi})$. Consider the general optimization problem, to minimise an objective non-linear function $S(\mathbf{V}_{\max};\phi)$ under linear inequality constraints.

$$\operatorname{Min} S(\mathbf{V}_{\max}; \boldsymbol{\phi}) = \operatorname{Min}\left\{ \sum_{i=1}^{N} [\mathbf{P}_i - F(v_{\max,i}; \boldsymbol{\phi})]^2 \right\} \tag{1}$$

where ϕ is a vector which contains the unknown parameters of the distribution function. $\mathbf{P}$ is a vector that contains the experimental cumulative relative frequencies. In other words, if the observed wind speed values are grouped into N wind speed intervals $0 - v_1, v_1 - v_2, \ldots, v_{N-1} - v_N$ and to each interval is assigned its relative frequency of occurrence fr_1, fr_2, ..., fr_N, then the cumulative frequencies will be given by: $P_1 = \mathrm{fr}_1$, $P_2 = P_1 + \mathrm{fr}_2$, $P_N = P_{N-1} + \mathrm{fr}_N$. The vector $\mathbf{V}_{\max}$ contains the maximum recorded wind speed values within each of the N intervals.

The distribution functions reviewed in this paper are not linear. However, those that can be expressed in closed form (Weibull and Rayleigh distributions) are susceptible to linearization. In the case of distribution functions which cannot be linearized, calculation of the parameters which minimise Eq. (1) can be performed through the use of a numerical method, such as that of Levenberg–Marquardt [208].

3.2. Considered statistical decisions

There exist various statistical tests to see whether a sample of wind data comes from a population with a particular probability distribution. However, in statistical analysis of wind the use of goodness of fit tests is not very common. The typically proposed or used goodness of fit tests have been as follows: the Chi-square test [10,42,43,47,92,131,162,194,204,205,209], the Kolmogorov–Smirnov test [101,105,117,157,186] and the Anderson–Darling test [151,106]. However, most often use has been made of a superimposed representation of the histogram of the sample of wind speeds and the fitted PDF to estimate, from a visual comparison, the goodness of fit.

Some authors use the coefficient of determination to measure the goodness of fit on an absolute scale. This coefficient is normally used to measure the linear correlation existing between the cumulative probabilities calculated with the fitted model and with the sample of wind speeds [20,42,43,50,47,92,149,153–155,189,201,204,205,208].

3.3. Notes on the methods and tests used

As pointed out by several authors [145,153,162,210–214], the wind speed recorded at short intervals of time usually presents dependence, and therefore the use of such samples violates the hypotheses on which the estimation techniques and the tests are based. In other words, as indicated by Ramírez and Carta [151], in theory, if the randomness assumption does not hold, then: (a) all of the usual statistical tests are invalid,[7] (b) the calculated uncertainties for commonly used statistics become meaningless and (c) the parameter estimates become suspect and non-supportable.

Ramírez and Carta [151] analysed the influence of the data sampling interval in the estimation of the parameters of the Weibull wind speed probability density distribution. According to them, the use of auto-correlated successive hourly mean wind speeds, though invalidating all of the usual statistical tests, has no significant effect on the shape of the probability density distribution.

[4] The parameters are a numerical characterisation of the population distribution which completely describes the probability density function.

[5] The classical approach supposes that the parameters are unknown fixed amounts about which no relevant initial information is available.

[6] From a Bayesian point of view a parameter is always a random variable with some type of probability distribution.

[7] Though some authors ignore this circumstance [105,106].

4. Review of mathematical/numerical modelling of PDFs

This section of the paper defines the probability density functions, $f(v)$, which have been used in the specialised literature on wind energy and other renewable energy sources. Also shown here is the estimation of the parameters using MM, MLM and LSM. With the aim of analysing the advantages of considering null wind speeds (calms) in the description of wind regimes, use is made of what was called by Takle and Brown [190] the hybrid distribution, $h(v, \boldsymbol{\phi}, \theta_0)$. The $h(v, \boldsymbol{\phi}, \theta_0)$ are related to the $f(v, \boldsymbol{\phi})$ through Eq. (2). The $h(v, \boldsymbol{\phi}, \theta_0)$ are defined in this paper[8] for all the $f(v, \boldsymbol{\phi})$ which do not pick up the frequency of null wind speed.

$$h(v, \boldsymbol{\phi}, \theta_0) = \theta_0 \delta(v) + (1 - \theta_0) f(v, \boldsymbol{\phi}) \tag{2}$$

where, θ_0 is the probability of observation of null wind speed. When $\theta_0 = 0$ the hybrid distribution $h(v, \boldsymbol{\phi}, \theta_0)$ matches the standard distribution $f(v, \boldsymbol{\phi})$. Where $\delta(v)$ is the Dirac operator and is given by the following equation:

$$\delta(v) = \begin{cases} 1 & \text{if } v = 0 \\ 0 & \text{if } v \neq 0 \end{cases} \tag{3}$$

The corresponding distribution function $H(v)$ of Eq. (2) is given, according to Takle and Brown [190] by the following equation:

$$H(v, \boldsymbol{\phi}, \theta_0) = \begin{cases} \theta_0 + (1 - \theta_0) F(v, \boldsymbol{\phi}) & \text{if } v \geq 0 \\ 0 & \text{if } v < 0 \end{cases} \tag{4}$$

The PDFs of power density, $\mathrm{wp}(v, \boldsymbol{\phi}, \theta_0)$, are obtained from the wind speed probability density functions through the following equation:

$$\mathrm{wp}(v, \boldsymbol{\phi}, \theta_0) = \frac{v^3 h(v, \boldsymbol{\phi}, \theta_0)}{\int_a^b v^3 h(v, \boldsymbol{\phi}, \theta_0)\,\mathrm{d}v} \tag{5}$$

where a and b represent the upper and lower bounds of the interval where the function $h(v, \boldsymbol{\phi}, \theta_0)$ [202] is defined.

4.1. Three parameter generalised gamma distribution

In accordance with Eq. (2), the hybrid GG.pdf is defined by the following equation:

$$h(v; \alpha, \beta, \eta, \theta_0) = \theta_0 \delta(v) + (1 - \theta_0) \frac{\alpha v^{\eta - 1}}{\beta^{-\eta/\alpha} \Gamma(\eta/\alpha)} \exp[-\beta v^{\alpha}] \tag{6}$$

where α and η are shape factors and $\beta^{-1/\alpha}$ is a scale factor [198]. $\Gamma(\cdot)$is the gamma function of Euler [215]. The cumulative distribution function is given by the following equation:

$$H(v; \alpha, \beta, \eta, \theta_0) = \theta_0 + (1 - \theta_0) \int_0^v \frac{\alpha v^{\eta - 1}}{\beta^{-\eta/\alpha} \Gamma(\eta/\alpha)} \exp[-\beta v^{\alpha}]\,\mathrm{d}v \tag{7}$$

4.1.1. Moment estimates of the parameters

In order to carry out the estimation of the three parameters of the generalised gamma distribution, we propose in this paper the use of the method suggested by Stacy and Mihram [216], which is based on the use of the first moments of the variable ln V.

Through the use of Eq. (8)[9] the relation η/α is determined. In Eq. (8) m_2 and m_3 are the variance and the third moment with respect to the mean of the napierian logarithms of the sample data and, therefore, null wind speeds are not considered. $\psi'(\eta/\alpha)$ is the trigamma function and $\psi''(\eta/\alpha)$ is the tetragamma function [215]. Eq. (7) can be resolved using a combination of the bisection method and the Newton–Raphson method [217].

$$\frac{m_3}{(m_2)^{3/2}} = \frac{\psi''(\eta/\alpha)}{[\psi'(\eta/\alpha)]^{3/2}} \tag{8}$$

Once the relation η/α is determined, the parameter α is obtained from the following equation:

$$\alpha = \frac{m_2}{m_3} \frac{\psi''(\eta/\alpha)}{\psi'(\eta/\alpha)} \tag{9}$$

Once η/α and α are known, the parameter η is obtained from the following equation:

$$\eta = \left(\frac{\eta}{\alpha}\right)\alpha \tag{10}$$

The parameter β is deduced from Eq. (11), where $\psi(\eta/\alpha)$ is the digamma function [215] and m is the mean of the napierian logarithms of the sample data

$$\beta = \exp\left[\psi\left(\frac{\eta}{\alpha}\right) - m\alpha\right] \tag{11}$$

4.1.2. Maximum likelihood estimates of the parameters

Operating with the maximum likelihood function [218] Eq. (12) can be obtained:

$$\alpha T_3 - \ln[\alpha(T_2 - T_1 T_3)] - \psi\left[\frac{T_1}{\alpha(T_2 - T_1 T_3)}\right] = 0 \tag{12}$$

where

$$T_1 = n^{-1} \sum_{i=1}^{n} (v_i)^{\alpha} \qquad T_2 = n^{-1} \sum_{i=1}^{n} (v_i)^{\alpha} \ln(v_i)$$
$$T_3 = n^{-1} \sum_{i=1}^{n} \ln(v_i) \tag{13}$$

In order to determine α Eq. (12) can be resolved using a combination of the bisection method and the Newton–Raphson method [217]. Then β and η are determined through the following equation:

$$\beta = \frac{1}{\alpha(T_2 - T_1 T_3)}, \qquad \eta = \frac{T_1}{T_2 - T_1 T_3} \tag{14}$$

4.2. Two parameter gamma distribution

In accordance with Eq. (2), the hybrid G.pdf is defined by the following equation:

$$h(v; \beta, \eta, \theta_0) = \theta_0 \delta(v) + (1 - \theta_0) \frac{v^{\eta - 1}}{\beta^{\eta} \Gamma(\eta)} \exp[-v/\beta] \tag{15}$$

The G.pdf is a particular case of the GG.pdf.

[8] According to the information available to the authors of this work, with the exception of Auwera et al. [10], the hybrid distribution of Takle and Brown [190] was exclusively generated with the two parameter Weibull distribution. Auwera et al. [10] propose the construction of hybrid distributions with lognormal and three parameter gamma generalised laws (hypergamma, modified gamma, Weibull three-parameter [10]) and laws derived from the latter: two parameter Weibull, one parameter Rayleigh, two parameter gamma (Pearson-type III), one parameter exponential and one parameter Chi-squared. Auwera et al. [10] estimate the parameters using the maximum likelihood method and check the goodness of fits using the χ^2-test.

[9] As can be seen, the parameters calculated with this procedure do not pick up the periods of null wind speeds.

The cumulative distribution function is given by the following equation:

$$H(v;\beta,\eta,\theta_0)=\theta_0+(1-\theta_0)\int_0^v \frac{v^{\eta-1}}{\beta^\eta\Gamma(\eta)}\exp\left[-\frac{v}{\beta}\right]dv \tag{16}$$

4.2.1. Moment estimates of the parameters

In order to carry out the estimation of the two unknown parameters, the mean and variance of the W.pdf [219] are equated to the corresponding values of the sample[10] (m and s^2). From the resolution of the proposed system of two equations the estimated values of the parameters are obtained (Eq. (17))

$$\beta=\frac{s^2}{m},\qquad \eta=\frac{m^2}{s^2} \tag{17}$$

4.2.2. Maximum likelihood of the parameters

Operating with the maximum likelihood function [219] Eq. (18) can be obtained

$$\ln(\eta)-\psi(\eta)=\ln\left(\frac{\sum_{i=1}^n v_i}{\sum_{i=1}^n \ln(v_i)}\right) \tag{18}$$

Eq. (18) can be resolved using a combination of the bisection method and the Newton–Raphson method [217].

Once parameter η is determined that satisfies Eq. (18), parameter β is obtained from the following equation:

$$\beta=\frac{1}{n\eta}\sum_{i=1}^n(v_i) \tag{19}$$

4.3. Two parameter Weibull distribution

In accordance with Eq. (2), the hybrid W.pdf is defined by the following equation:

$$h(v;\alpha,\beta,\theta_0)=\theta_0\delta(v)+(1-\theta_0)\left(\frac{\alpha}{\beta}\right)\left(\frac{v}{\beta}\right)^{\alpha-1}\exp\left[\left(\frac{-v}{\beta}\right)^\alpha\right] \tag{20}$$

The cumulative distribution function is given by the following equation:

$$H(v;\alpha,\beta,\theta_0)=\theta_0+(1-\theta_0)\left[1-\exp\left[-\left(\frac{v}{\beta}\right)\right]^\alpha\right] \tag{21}$$

4.3.1. Moment estimates of the parameters

In order to carry out the estimation of the two unknown parameters, the mean and variance of the W.pdf [219] are equated to the corresponding values of the sample[11] (m and s^2). From the resolution of the proposed system of two equations the estimated values of the parameters are obtained.[12] According to the authors of this paper, the shape parameter α can be estimated through the approximate expression given by the following equation:

$$\alpha=\left(\frac{s}{m}\right)^{-1.091} \tag{22}$$

The scale parameter β is determined from the following equation [1]:

$$\beta=\frac{m}{\Gamma(1+1/\alpha)} \tag{23}$$

4.3.2. Maximum likelihood estimates of the parameters

Operating with the maximum likelihood function [219] Eq. (24) can be obtained:

$$\alpha=\left[\frac{\sum_{i=1}^n v_i^\alpha \ln v_i}{\sum_{i=1}^n v_i^\alpha}+\frac{1}{n}\sum_{i=1}^n \ln(v_i)\right]^{-1} \tag{24}$$

In order to estimate α, Eq. (24) can be resolved using an iterative procedure. Parameter β is determined from the following equation:

$$\beta=\left(\frac{1}{n}\sum_{i=1}^n(v_i)^\alpha\right)^{1/\alpha} \tag{25}$$

4.3.3. Least square estimates of the parameters

The shape parameter α, as shown by Justus et al. [67], can be estimated through the following equation:

$$\alpha=\left\{N\sum_{i=1}^N Y_iX_i-\left(\sum_{i=1}^N X_i\right)\left(\sum_{i=1}^N Y_i\right)\right\}\left\{N\sum_{i=1}^N X_i^2-\left(\sum_{i=1}^N X_i\right)^2\right\}^{-1} \tag{26}$$

where

$$Y_i=\ln\{-\ln[1-P_i]\}\qquad X_i=\ln v_{\max,i} \tag{27}$$

In Eq. (27), the P_i are the components of the vector $\mathbf{P}$, which contains the experimental cumulative relative frequencies of the N intervals in which the n wind speeds of the sample have been classified. $v_{\max,i}$ are the maximum recorded wind speed values within each of the N intervals (see Section 3.1). The scale parameter β is estimated from the following equation:

$$\beta=\exp\left[-\frac{k}{\alpha}\right] \tag{28}$$

where k is estimated through the following equation:

$$k=\left\{\sum_{i=1}^N Y_i\sum_{i\,i=1}^N X_i^2-\left(\sum_{i=1}^N X_i\right)\left(\sum_{i=1}^N X_iY_i\right)\right\}\left\{N\sum_{i=1}^N X_i^2-\left(\sum_{i=1}^N X_i\right)^2\right\}^{-1} \tag{29}$$

4.4. One parameter Rayleigh distribution

In accordance with Eq. (2), the hybrid R.pdf is defined by the following equation:

$$h(v;\theta,\theta_0)=\theta_0\delta(v)+(1-\theta_0)\left(\frac{v}{\theta^2}\right)\exp\left[-\left(\frac{v}{\sqrt{2}\theta}\right)^2\right] \tag{30}$$

The R.pdf is a particular case of the W.pdf when $\alpha = 2$. The cumulative distribution function is given by the following equation:

$$H(v;\theta,\theta_0)=\theta_0+(1-\theta_0)\left[1-e^{-v^2/(2\theta^2)}\right] \tag{31}$$

[10] Once the null wind speeds are eliminated, if dealing with the hybrid distribution. With all the values, if dealing with the standard distribution.

[11] Once the null wind speeds are eliminated, if dealing with the hybrid distribution. With all the values, if dealing with the standard distribution.

[12] Another method which could be catalogued as a moments method, and which has also been proposed for estimation of the shape parameter α, consists of equating the so-called energy pattern factor [2,66,176–178] or cube factor [2,4] of the PDF with the corresponding factor of the wind speed data sample. The cube factor describes the ratio of the mean of wind speed cubed to the cube of the mean wind speed. Once α is estimated, the scale parameter β is estimated using Eq. (23).

4.4.1. Moment estimator of the parameter

In order to carry out the estimation of the unknown parameter, the mean of the distribution is equated [219] to the corresponding mean of the distribution of the sample[13] (m). From this equality the parameter θ is obtained, Eq. (32)

$$\theta = \frac{\sqrt{2}}{\sqrt{\pi}} m \tag{32}$$

4.4.2. Maximum likelihood estimator of the parameter

Operating with the maximum likelihood function [219], Eq. (33) can be obtained, which enables estimation of θ

$$\theta = \left(\frac{1}{2n} \sum_{i=1}^{n} (v_i)^2 \right)^{1/2} \tag{33}$$

4.4.3. Least square estimator of the parameter

The parameter θ can be estimated through the following equation:

$$\theta = \left\{ \frac{1}{2} \exp\left[-\frac{1}{N} \left(\sum_{i=1}^{N} Y_i - 2\sum_{i=1}^{N} X_i \right) \right] \right\}^{1/2} \tag{34}$$

where X_i and Y_i are given by Eq. (27).

4.5. Two parameter square-root normal distribution (SRN.pdf)

In accordance with Eq. (2), the hybrid SRN.pdf is defined by the following equation:

$$h(v; \mu, \sigma, \theta_0) = \theta_0 \delta(v) + (1 - \theta_0) \frac{1}{\sigma\sqrt{8\pi v}} \exp\left[\frac{-(\sqrt{v} - \mu)^2}{2\sigma^2} \right] \tag{35}$$

where μ and σ are the mean and standard deviation of the distribution.

The cumulative distribution function is given by the following equation:

$$H(v; \mu, \sigma, \theta_0) = \theta_0 + (1 - \theta_0) \int_0^{\infty} \frac{1}{\sigma\sqrt{8\pi v}} \exp\left[\frac{-(\sqrt{v} - \mu)^2}{2\sigma^2} \right] dv \tag{36}$$

4.5.1. Moment estimates of the parameters

In this case, the mean and variance of the SRN.pdf distribution are equated to the corresponding values of the sample[14] (m and s^2).

$$\mu = m = \frac{\sum_{i=1}^{n} v_i^{1/2}}{n}, \qquad \sigma^2 = s^2 = \frac{\sum_{i=1}^{n} \left(v_i^{1/2} - m\right)^2}{n} \tag{37}$$

4.5.2. Maximum likelihood estimates of the parameters

Operating with the maximum likelihood function, the equalities shown in Eq. (37) are obtained as a result.

[13] Once the null wind speeds are eliminated, if dealing with the hybrid distribution. With all the values, if dealing with the standard distribution.

[14] Once the null wind speeds are eliminated, if dealing with the hybrid distribution. With all the values, if dealing with the standard distribution.

4.6. Two parameter normal truncated distribution (NT.pdf)

The singly truncated from below normal distribution is defined by the following equation [155,198]:

$$f(v; \alpha, \beta) = \frac{1}{I_0(\alpha, \beta)\beta\sqrt{2\pi}} \exp\left[\frac{-(v - \alpha)^2}{2\beta^2} \right] \tag{38}$$

where $I_0(\alpha,\beta)$ is expressed by the following equation:

$$I_0(\alpha, \beta) = \frac{1}{\beta\sqrt{2\pi}} \int_0^{\infty} \exp\left[\frac{-(v - \alpha)^2}{2\beta^2} \right] dv \tag{39}$$

The cumulative distribution function is given by the following equation:

$$F(v; \alpha, \beta) = \int_0^{v} \frac{1}{I_0(\alpha, \beta)\beta\sqrt{2\pi}} e^{-(v-\alpha)^2/(2\beta^2)} dv \tag{40}$$

4.6.1. Moment estimates of the parameters

In order to carry out estimation of the two unknown parameters (α and β) of the distribution, the first three statistical moments of the sample with respect to the origin (m, m'_2, m'_3)[15] can be used. These are equated to the first three moments with respect to the origin of the distribution, as proposed by Cohen [220]. As shown by Cohen [220], the following expressions are obtained of the estimated parameters, Eq. (41)

$$\alpha = \frac{2m'_2 m - m'_3}{2m^2 - m'_2}, \qquad \beta^2 = \frac{m m'_3 - m'^2_2}{2m^2 - m'_2} \tag{41}$$

4.6.2. Maximum likelihood estimates of the parameters

Operating with the maximum likelihood function [198] Eq. (42) can be obtained

$$\left[\frac{Z(\zeta)}{1 - \phi(\zeta)} - \zeta \right]^{-1} \left\{ \left[\frac{Z(\zeta)}{1 - \phi(\zeta)} - \zeta \right]^{-1} - \zeta \right\} = \frac{1}{n(m)^2} \sum_{i=1}^{n} v_i^2 \tag{42}$$

where

$$\zeta = -\frac{\alpha}{\beta} \tag{43}$$

$$Z(\zeta) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\zeta^2}{2} \right], \qquad 1 - \phi(\zeta) = \frac{1}{\sqrt{2\pi}} \int_0^{\zeta} \exp\left[-\frac{x^2}{2} \right] dx \tag{44}$$

Eq. (42) is known as the Pearson–Lee–Fisher equation [220], and can be resolved, to determine ζ, using a combination of the bisection method and the Newton–Raphson method [217]. β is estimated with Eq. (45) and α with Eq. (43)

$$\beta = \left(\frac{Z(\zeta)}{1 - \phi(\zeta)} - \zeta \right)^{-1} m \tag{45}$$

[15] The singly truncated from below normal distribution takes into account null wind speeds. Therefore, the three moments with respect to the origin of the sample pick up all the wind speed values, including null wind speeds.

4.7. Two parameter lognormal distribution (LN.pdf)

In accordance with Eq. (2), the hybrid LN.pdf is defined by the following equation:

$$h(v;\alpha,\beta,\theta_0)=\theta_0\delta(v)+(1-\theta_0)\frac{1}{v\beta\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left[\frac{\ln(v)-\alpha}{\beta}\right]^2\right\} \tag{46}$$

The standard LN.pdf depends on two parameters (α and β) [219] and does not take into account null wind speeds. The cumulative distribution function is given by the following equation:

$$H(v;\alpha,\beta,\theta_0)=\theta_0+(1-\theta_0)\int_{>0}^{v}\frac{1}{v\beta\sqrt{2\pi}}\,e^{-\{(1/2)[(\ln(v)-\alpha)/\beta]^2\}}\,dv \tag{47}$$

4.7.1. Moment estimates of the parameters

In order to carry out the estimation of the two unknown parameters of the distribution, the sample mean and variance[16] can be equated to the mean and variance of the distribution. Resolving the resulting system of two equations with two unknowns, the estimated values of α and β are obtained, Eq. (48).

$$\alpha=\ln\left[\frac{m}{\sqrt{1+(s^2/m^2)}}\right],\qquad \beta=\sqrt{\ln\left[1+\frac{s^2}{m^2}\right]} \tag{48}$$

4.7.2. Maximum likelihood estimates of the parameters

Operating with the maximum likelihood function [219], Eq. (49) can be obtained, which enables estimation of the values of the parameters α and β.

$$\alpha=\frac{1}{n}\sum_{i=1}^{n}\ln(v_i),\qquad \beta^2=\frac{1}{n}\sum_{i=1}^{n}[\ln(v_i)-\alpha]^2 \tag{49}$$

4.8. Two parameter inverse Gaussian distribution

In accordance with Eq. (2), the hybrid GI.pdf is defined by the following equation:

$$h(v;\alpha,\beta,\theta_0)=\theta_0\delta(v)+(1-\theta_0)\left(\frac{\beta}{2\pi v^3}\right)^{1/2}\exp\left[-\frac{\beta(v-\alpha)^2}{2v\alpha^2}\right] \tag{50}$$

The standard IG.pdf depends on two parameters (α and β) [198] and does not take into account null wind speeds. The cumulative distribution function is given by the following equation:

$$H(v;\alpha,\beta,\theta_0)=\theta_0+(1-\theta_0)\int_{0}^{v}\left[\frac{\beta}{2\pi v^3}\right]^{1/2}\exp\left[-\frac{\beta(v-\alpha)^2}{2v\alpha^2}\right]dv \tag{51}$$

4.8.1. Moment estimates of the parameters

In order to carry out the estimation of the two unknown parameters (α and β) of the distribution, the sample mean and variance (m, s^2)[17] can be equated to the mean and variance of the distribution. Resolving the resulting system of two equations with two unknowns, the estimated values of α and β are obtained, Eq. (52).

$$\alpha=m,\qquad \beta=\frac{m^3}{s^2} \tag{52}$$

4.8.2. Maximum likelihood estimates of the parameters

Operating with the maximum likelihood function [219] the values of the parameters α and β can be obtained, Eq. (53).

$$\alpha=\frac{1}{n}\sum_{i=1}^{n}v_i,\qquad \beta=n\left\{\sum_{i=1}^{n}v_i^{-1}-n^2\left(\sum_{i=1}^{n}(v_i)\right)^{-1}\right\}^{-1} \tag{53}$$

4.9. Three parameter beta distribution (B.pdf)

In accordance with Eq. (2), the hybrid B.pdf is defined by the following equation:

$$h(v;\alpha,\beta,\xi,\theta_0)=\theta_0\delta(v)+(1-\theta_0)\frac{1}{\xi}\frac{1}{B(\alpha,\beta)}\left(\frac{v}{\xi}\right)^{\alpha-1}\left(\frac{\xi-v}{\xi}\right)^{\beta-1} \tag{54}$$

where $B(\alpha,\beta)$ is the beta function of Euler [215], α and β are the shape parameters (both positive), and ξ is a position parameter [219]. The cumulative distribution function is given by the following equation:

$$H(v;\alpha,\beta,\xi,\theta_0)=\theta_0+(1-\theta_0)\int_0^v\frac{1}{\xi}\frac{1}{B(\alpha,\beta)}\left(\frac{v}{\xi}\right)^{\alpha-1}\left(\frac{\xi-v}{\xi}\right)^{\beta-1}dv \tag{55}$$

4.9.1. Moment estimates of the parameters

In order to carry out the estimation of the three parameters (α, β and ξ) of the B.pdf the first three statistical moments of the sample can be equated to the corresponding first three moments of the distribution [219]. As a result of this procedure a non-linear system equation has to be solved.[18] However, in this paper it is proposed that the first and second moment of the distribution are equated to the corresponding moments of the sample (m, m_2'). The parameter ξ is directly estimated, equating it to the maximum value of the speed ($v_{\max}$) of the sample. This entails considering that the probability that wind speeds greater than $v_{\max}$ occur in the population is null. The larger the size of the sample the greater the validity of this hypothesis.

$$\xi=v_{\max},\qquad \alpha=\frac{1}{\xi}\left(\frac{m^2\xi-mm_2'}{m_2'-m^2}\right),\qquad \beta=\left(\frac{m\xi-m_2'}{m_2'-m^2}\right)-\alpha \tag{56}$$

4.9.2. Maximum likelihood estimates of the parameters

Operating with the maximum likelihood function [219] we can obtain Eq. (57). The resolution of the non-linear system of equations Eq. (57) enables estimation of the values of the parameters α and β.

$$\begin{aligned}\psi(\beta)-\psi(\alpha+\beta)&=\frac{1}{n}\sum_{i=1}^{n}\ln\left[1-v_i\exp\left[\psi(\alpha)-\psi(\alpha+\beta)-\frac{1}{n}\sum_{i=1}^{n}\ln v_i\right];\frac{\alpha+\beta-1}{\beta-1}\right.\\&=\frac{1}{n}\sum_{i=1}^{n}\left[1-v_i\exp\left[\psi(\alpha)-\psi(\alpha+\beta)-\frac{1}{n}\sum_{i=1}^{n}\ln v_i\right]\right]^{-1}\end{aligned} \tag{57}$$

[16] Once the null wind speeds are eliminated, if dealing with the hybrid distribution. With all the values, if dealing with the standard distribution.

[17] Once the null wind speeds are eliminated, if dealing with the hybrid distribution. With all the values, if dealing with the standard distribution.

[18] The experience of the authors of this paper is that these solutions usually result in the restriction imposed on the position parameter $\xi \geq v_{\max}$ not being met.

Table 1
Descriptive numerical measurements of the four weather stations analysed

Weather stations	Geographical coordinates		Height (m)	Years	Mean, m (m s^{-1})	Variance, s^2 (m^2 s^{-2})	Calms, θ_0 (%)	Maximum, v_{max} (m s^{-1})	Skewness, Sk	Kurtosis, Kt	Median, Me (m s^{-1})
	Latitude	Length									
La Palma	28°37′13″N	17°45′13.1W	10	2003–2005	4.61	4.55	6.615×10^{-3}	22.00	0.55	3.89	4.20
Granadilla	28°4′5.7″N	16°30′26.3″W	10	1998–2000, 2002, 2004	6.95	15.04	1.439×10^{-3}	19.8	0.254	2.12	6.80
P. Gorra	27°57′36″N	15°33′35″W	20	2005–2007	6.40	18.09	0.014	32.3	0.946	3.93	5.80
Amagro	28°7′58.9″N	15°40′55″W	20	1997–1999, 2001–2003, 2005, 2007	7.89	15.11	4.625×10^{-3}	24.30	0.115	2.48	8.0

The parameter ξ is then estimated using the following equation:

$$\xi = \exp\left[\psi(\alpha+\beta) - \psi(\alpha) + \frac{1}{n}\sum_{i=1}^{n}\ln v_i\right] \quad (58)$$

4.10. Two components mixture Weibull distribution (MWW.pdf)

In accordance with Eq. (2), the hybrid MWW.pdf is defined by the following equation:

$$h(v;\alpha_1,\beta_1,\alpha_2,\beta_2,\omega,\theta_0) = \theta_0\delta(v) + (1-\theta_0)\left\{\frac{\omega\alpha_1}{\beta_1(v/\beta_1)^{\alpha_1-1}}\exp\left[\left(\frac{-v}{\beta_1}\right)^{\alpha_1}\right] + (1-\omega)\frac{\alpha_2}{\beta_2(v/\beta_2)^{\alpha_2-1}}\exp\left[\left(\frac{-v}{\beta_2}\right)^{\alpha_2}\right]\right\} \quad (59)$$

where $0\le\omega\le1$ is a mixture parameter, α_1 and α_2 are shape parameters and β_1 and β_2 are scale parameters.

The cumulative distribution function is given by the following equation:

$$H(v;\alpha_1,\beta_1,\alpha_2,\beta_2,\omega,\theta_0) = \theta_0 + (1-\theta_0)[\omega[1-e^{-(v/\beta_1)^{\alpha_1}}] + (1-\omega)[1-e^{-(v/\beta_2)^{\alpha_2}}]] \quad (60)$$

4.10.1. Moment estimates of the parameters

In order to estimate the five parameters of the hybrid MWW.pdf, the first five statistical moments of the theoretical distribution are equated to the corresponding moments of the sample, $m'_r(r=1,\ldots,5)$.[19]

$$m'_r = \omega\beta_1^r\Gamma\left(1+\frac{r}{\alpha_1}\right) + (1-\omega)\beta_2^r\Gamma\left(1+\frac{r}{\alpha_2}\right) \quad (61)$$

The system of Eq. (61) does not have an analytical solution. In this paper we propose the use of a Quasi–Newton algorithm [217] to numerically resolve this system of equations. However, it should be pointed out that there are, in general, a number of potential problems related to the application of the method of moments in mixture distributions [221].

4.10.2. Maximum likelihood estimates of the parameters

In order to find the values of the parameters (α_1, β_1, α_2, β_2 and ω) the likelihood function or the logarithm of the likelihood function has to be maximised, Eq. (62).

$$\text{maximizar}\ \ln L(v_i;\alpha_1,\beta_1,\alpha_2,\beta_2,\omega) = \sum_{i=1}^{m}\ln\left\{\omega\left[\frac{\alpha_1}{\beta_1}\left(\frac{v_i}{\beta_1}\right)^{\alpha-1}\exp\left[-\left(\frac{v_i}{\beta_1}\right)^{\alpha_1}\right]\right] + (1-\omega)\left[\frac{\alpha_2}{\beta_2}\left(\frac{v_i}{\beta_2}\right)^{\alpha_2-1}\exp\left[-\left(\frac{v_i}{\beta_2}\right)^{\alpha_2}\right]\right]\right\} \quad (62)$$

Various numerical methods have been proposed to resolve Eq. (62) [221]. Amongst the most familiar should be mentioned the Newton–Raphson method (NR), the Scoring method (SM) and the expectation-maximization (EM) algorithm. Each of these has a series of advantages and drawbacks [221].

4.11. Singly truncated normal Weibull mixture distribution (MTNW.pdf)

In accordance with Ref. [155] the MTNW.pdf is defined by the following equation:

$$f(v;\alpha_1,\beta_1,\alpha,\beta,\omega) = \frac{\omega\alpha_1}{\beta_1(v/\beta_1)^{\alpha_1-1}}\exp\left[\left(\frac{-v}{\beta_1}\right)^{\alpha_1}\right] + (1-\omega)\frac{1}{I_0(\alpha,\beta)\beta\sqrt{2\pi}}\exp\left[\frac{-(v-\alpha)^2}{2\beta^2}\right] \quad (63)$$

where $I_0(\alpha,\beta)$is expressed by Eq. (39).

The cumulative distribution function is given by the following equation:

$$F(v;\alpha_1,\beta_1,\alpha,\beta,\omega) = \omega[1-e^{-(v/\beta_1)^{\alpha_1}}] + (1-\omega)\int_0^v \frac{1}{I_0(\alpha,\beta)\beta\sqrt{2\pi}}\exp\left[\frac{-(v-\alpha)^2}{2\beta^2}\right]dv \quad (64)$$

4.11.1. Moment estimates of the parameters

In order to estimate the five parameters (α, β, α_1, β_1 and ω) of the MTNW.pdf the first five statistical moments of the theoretical distribution are equated to the corresponding moments of the sample $m'_r(r=1,\ldots,5)$.

$$m'_r = \omega\beta_1^r\Gamma\left(1+\frac{r}{\alpha_1}\right) + (1-\omega) + (1-\omega)\int_0^\infty \frac{v^r}{I_0(\alpha,\beta)\beta\sqrt{2\pi}}\exp\left[\frac{-(v-\alpha)^2}{2\beta^2}\right]dv \quad (65)$$

The system of Eq. (65) can be resolved using the techniques indicated in Section 4.10.1

4.11.2. Maximum likelihood estimates of the parameters

In order to find the values of the parameters (α, β, α_1 β_1 and ω) the likelihood function or the logarithm of the likelihood function has to be maximised, Eq. (66).

[19] Once the null wind speeds are eliminated, if dealing with the hybrid distribution. With all the values, if dealing with the standard distribution.

Table 2
La Palma weather station

Method	GG.pdf			G.pdf		W.pdf		NT.pdf		WW.pdf					R.pdf	B.pdf			NR.pdf		IG.pdf		LN.pdf		WNT.pdf				
	α	β	η	β	η	α	β	α	β	α_1	β_1	α_2	β_2	ω	θ	α	β	ξ	μ	σ	α	β	α	β	α	β	α_1	β_1	ω
MM	1.7792	0.0918	2.8642	0.955	4.861	2.316	5.241	4.457	2.338	3.159	5.391	1.624	4.803	0.651	3.705	3.638	13.599	22	2.097	0.495	4.644	22.575	1.442	0.433	3.861×10^{-6}	5.22	2.801	5.303	0.184
ML	1.812	0.083	2.817	1.06	4.383	2.343	5.244	4.48	2.272	2.401	5.222	3.083	13.478	0.9973	3.605	3.671	16.729	25.788	2.097	0.495	4.644	14.748	1.417	0.519	0	5.355	2.492	5.252	0.079
LS	2.369	0.018	1.991	1.165	3.822	2.128	4.942	4.08	2.378	2.249	2.995	2.977	5.851	0.34	3.491	3.45	16.751	25.77	2.029	0.54	4.536	16.403	1.387	0.505	4.871	2.124	2.428	2.851	0.75

Parameters of the standard probability density distributions (PDFs) estimated with moments (MM), maximum likelihood (ML) and least square (LS) methods.

Table 3
Granadilla weather station Parameters of the standard probability density functions (PDFs) estimated with moments (MM), maximum likelihood (ML) and least square (LS) methods

Method	GG.pdf			G.pdf		W.pdf		NT.pdf		WW.pdf					R.pdf	B.pdf			NR.pdf		IG.pdf		LN.pdf		WNT.pdf				
	α	β	η	β	η	α	β	α	β	α_1	β_1	α_2	β_2	ω	θ	α	β	ξ	μ	σ	α	β	α	β	α	β	α_1	β_1	ω
MM	4.3848	1.4251×10^{-5}	1.2438	2.155	3.228	1.937	7.845	6.243	4.417	3.241	9.995	2.109	3.247	0.671	5.551	1.823	3.418	20	2.529	0.749	6.958	22.46	1.805	0.519	3.02	1.752	3.474	10.335	0.375
ML	4.752	5.313×10^{-6}	1.281	2.832	2.457	1.831	7.812	6.168	4.523	2.102	3.521	3.434	10.235	0.3692	5.63	2.433	166.988	484.031	2.529	0.749	6.958	7.915	1.723	0.752	2.852	1.463	3.335	10.148	0.341
LS	4.295	1.676×10^{-5}	1.196	2.749	2.626	1.759	7.981	5.897	4.948	2.241	3.37	3.35	10.106	0.351	5.68	1.612	2.947	19.8	2.536	0.827	7.409	17.388	1.819	0.616	2.811	1.453	3.317	10.119	0.34

Table 4
P. Gorra weather station

Method	GG.pdf			G.pdf		W.pdf		NT.pdf		WW.pdf					R.pdf	B.pdf			NR.pdf		IG.pdf		LN.pdf		WNT.pdf				
	α	β	η	β	η	α	β	α	β	α_1	β_1	α_2	β_2	ω	θ	α	β	ξ	μ	σ	α	β	α	β	α	β	α_1	β_1	ω
MM	1.6161	0.0386	1.5415	4.861	0.955	1.576	7.224	3.651	6.022	1.863	8.649	1.265	5.056	0.574	5.175	1.702	7.173	33.823	2.4	0.851	6.486	15.366	1.694	0.593	3.176	6.705	2.192	5.95	0.741
ML	1.323	0.105	1.768	3.0	2.162	1.589	7.242	3.96	5.804	1.085	4.696	1.589	7.242	0	5.47	1.885	10.998	44.337	2.4	0.851	6.486	7.997	1.621	0.771	3.881	6.544	1.949	5.724	0.678
LS	1.604	0.04	1.528	3.009	2.18	1.559	7.187	4.076	5.555	1.493	6.361	1.789	8.59	0.629	5.1	1.862	11.002	44.741	2.381	0.861	6.764	13.049	1.693	0.67	2.406	4.225	1.759	8.642	0.428

Parameters of the standard probability density distributions (PDFs) estimated with moments (MM), maximum likelihood (ML) and least square (LS) methods.

Table 5
Amagro weather station

Method	GG.pdf			G.pdf		W.pdf		NT.pdf		WW.pdf					R.pdf	B.pdf			NR.pdf		IG.pdf		LN.pdf		WNT.pdf				
	α	β	η	β	η	α	β	α	β	α_1	β_1	α_2	β_2	ω	θ	α	β	ξ	μ	σ	α	β	α	β	α	β	α_1	β_1	ω
MM	4.47	1.0162×10^{-5}	1.5402	1.879	4.218	2.231	8.949	7.557	4.166	3.19	10.642	1.723	5.973	0.619	6.324	2.683	5.668	24.668	2.4	0.851	6.486	15.366	1.694	0.593	3.176	6.705	2.192	5.95	0.741
ML	4.23	1.953×10^{-5}	1.541	2.722	2.911	2.106	8.896	7.525	4.241	1.451	5.453	3.074	10.299	0.3007	6.233	2.878	146.079	409.754	2.728	0.696	7.926	8.643	1.889	0.719	5.944	4.537	3.7	11.08	0.654
LS	4.753	4.87×10^{-6}	1.538	2.089	3.924	2.171	9.093	7.541	4.32	1.568	6.668	3.491	10.567	0.456	6.417	2.719	6.061	26.013	2.755	0.728	8.35	31.451	2.002	0.496	2.536	2.179	2.987	10.115	0.194

Parameters of the standard probability density functions (PDFs) estimated with moments (MM), maximum likelihood (ML) and least square (LS) methods.

Table 6
Parameters of the maximum entropy probability density distributions (1: La Palma; 2: Granadilla; 3: P. Gorra; 4: Amagro)

	ME_5.pdf						ME_2.pdf			ME_4.pdf					ME_3.pdf			
	λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_0	λ_1	λ_2	λ_0	λ_1	λ_2	λ_3	λ_4	λ_0	λ_1	λ_2	λ_3
1	−3.358	0.78	−0.099	6.927×10^{-4}	3.431×10^{-5}	1.183×10^{-6}	−3.659	0.868	−0.097	−3.494	0.831	−0.103	7.375×10^{-4}	3.353×10^{-5}	−3.96	1.148	−0.162	4.147×10^{-3}
2	−2.946	0.155	−0.017	7.959×10^{-4}	-3.509×10^{-6}	-3.151×10^{-6}	−3.257	0.297	−0.024	−2.9	0.088	2.077×10^{-4}	-1.708×10^{-4}	-4.279×10^{-5}	−2.965	0.105	4.541×10^{-3}	-1.157×10^{-3}
3	−2.657	0.144	−0.018	5.769×10^{-5}	2.321×10^{-6}	1.183×10^{-6}	−2.626	0.118	−0.015	−2.575	0.112	−0.016	5.241×10^{-5}	2.082×10^{-6}	−3.554	0.224	-1.323×10^{-3}	-9.982×10^{-4}
4	−3.649	0.292	−0.014	-1.742×10^{-4}	-9.802×10^{-6}	-3.872×10^{-7}	−3.898	0.418	−0.028	−3.601	0.264	−0.01	-2.952×10^{-4}	−3.601	−3.554	0.224	-1.323×10^{-3}	-9.982×10^{-4}

$$\text{maximizar}\ln L(v_i;\alpha_1,\beta_1,\alpha,\beta,\omega) = \sum_{i=1}^{n}\ln\left\{\omega\left[\frac{\alpha_1}{\beta_1}\left(\frac{v_i}{\beta_1}\right)^{\alpha-1}\exp\left[-\left(\frac{v_i}{\beta_1}\right)^{\alpha_1}\right]\right] + (1-\omega)\left[\frac{1}{I_0(\alpha,\beta)\beta\sqrt{2\pi}}\exp\left[\frac{-(v_i-\alpha)^2}{2\beta^2}\right]\right]\right\} \quad (66)$$

In order to resolve Eq. (66) various methods can be used [221].

4.12. Maximum entropy probability density function (ME_M.pdf)

A probability density function $f(v)$ which is defined in the interval a,b and which is obtained by maximising Shannon's entropy [222] subject to the following restrictions (a) and (b), has the form given by Eq. (67) [223]. The restrictions are that: (a) the probability density function $f(v)$ must be such that the area below the curve, in the interval [a,b], is equal to one, (b) that the M low-order statistical moments with respect to the origin of the theoretical distribution must be equal to the M low-order statistical moments with respect to the origin, $m'_j(j=1,\ldots,M)$, determined numerically from the sample

$$f(v;\lambda_1,\ldots,\lambda_M) = \exp\left[\lambda_0 + \sum_{i=1}^{M}\lambda_i v^i\right] \quad (67)$$

where the parameter λ_0 is related to the M remaining parameters of the distribution by the following equation:

$$\lambda_0 = -\ln\left[\int_a^b \exp\left(\sum_{i=1}^{M}\lambda_i v^i\right)\mathrm{d}v\right] \quad (68)$$

The corresponding cumulative distribution function of the maximum entropy probability density function is given by the following equation:

$$F(v;\lambda_1,\ldots,\lambda_M) = \int_a^v \exp\left(\lambda_0 + \sum_{j=1}^{M}\lambda_j v^j\right)\mathrm{d}v \quad (69)$$

4.12.1. Moment estimates of the parameters

Eq. (70) represents M simultaneous non-linear equations that represent the restriction (b) indicated in Section 4.12. Eq. (70) has to be resolved in order to determine, using information provided from the sample, the $\lambda_1,\lambda_2,\ldots,\lambda_M$ distribution parameters [154]. The system of Eq. (70) has no analytical solution except for $M = 1$, and therefore it has to be resolved numerically. The parameter λ_0, which will take the non-null probability of calms, will be determined from Eq. (68).

$$\left[\int_a^b v^j\exp\left(\sum_{i=1}^{M}\lambda_i v^i\right)\mathrm{d}v\right]\left[\int_a^b \exp\left(\sum_{i=1}^{M}\lambda_i v^i\right)\mathrm{d}v\right]^{-1} = m'_j,\quad j=1,\ldots,M \quad (70)$$

In theory, the number of moments M that can be used is not limited, but if the sample size becomes too small, the estimates of the higher moments will tend to become meaningless. Siddall [223] has analysed the ability of the method to reproduce known distributions. According to Siddall [223], the indications are that quite good density functions can be generated for most shapes using four or five moments. Most authors [204,205] consider Eq. (67) defined in the interval $a = v_{\min}$ and $b = v_{\max}$, where $v_{\min}$ and $v_{\max}$ are the minimum and maximum speeds of the data sample. This entails supposing that the probability that there can be wind speeds outside this interval is null. In order to eliminate this restriction, we consider in this paper, as did Ramírez and Carta

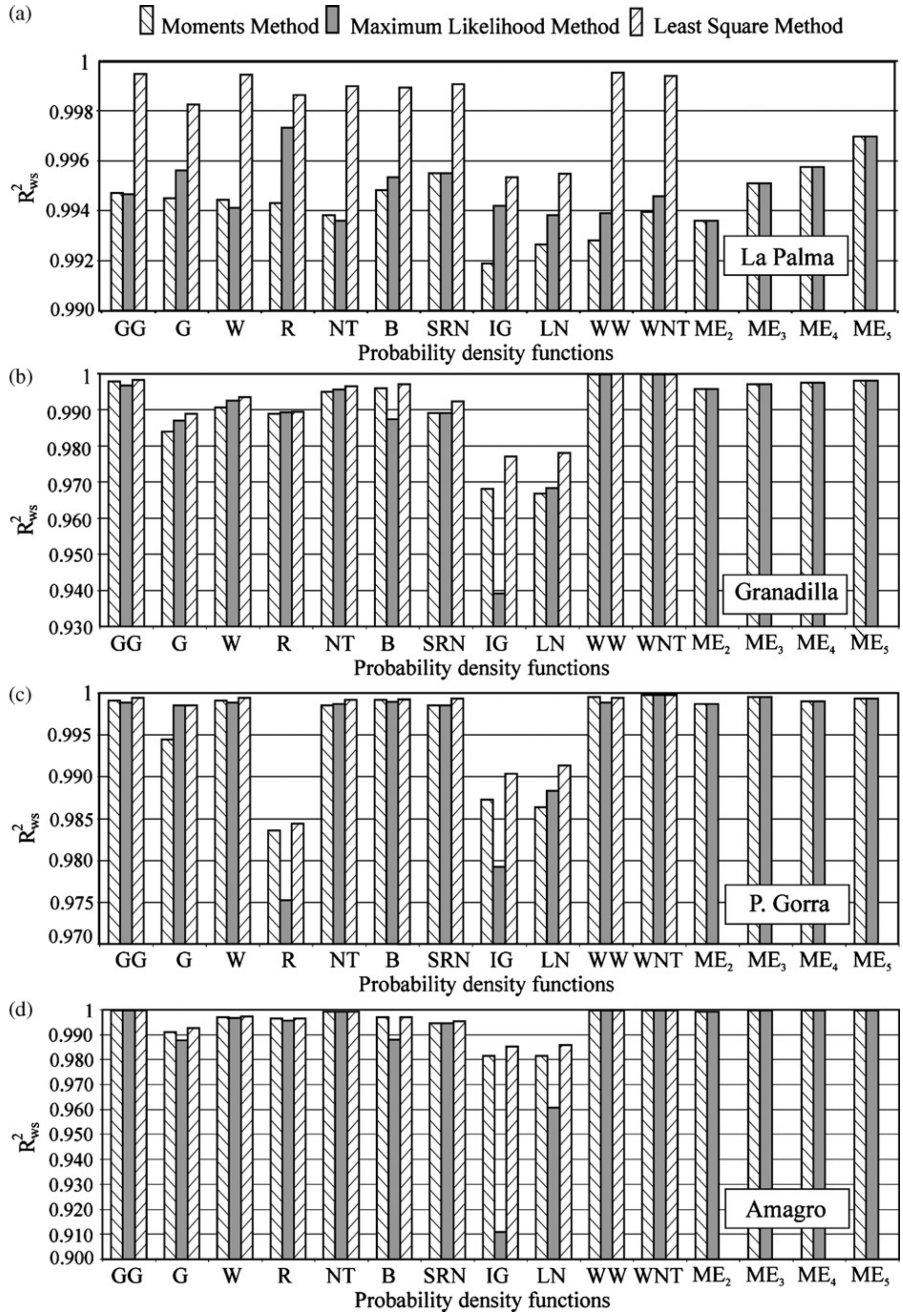

Fig. 1. Coefficients of determination R^2_{ws} obtained with the different non-hybrid PDFs analysed.

[154], that the margin of variation of the wind speed is in the interval $0 \le v \le \infty$.

5. Wind speeds used

In order to determine the maximum wind sourced energy potential that can be captured in the Canary Islands (Spain), a significant number of anemometer stations have been installed [206].

As a consequence of its geographical location,[20] the Canary Archipelago is regularly hit by trade winds from the north-east. However, the orographic and topographic characteristics of the islands and the effects of the sea breezes in certain areas give rise to the existence of different wind speed distributions in the same island.

In order to compare the different models included in the collection described in Section 3, four weather stations installed in the islands have been selected. The wind speeds recorded at these stations present different wind speed frequency histograms. These histograms, generated from the hourly mean wind speeds, can be considered to be representative of a significant number of weather stations.

Table 1 shows the number of stations, the geographical coordinates, the heights above ground level, the data recording period, the mean (m), variance (s^2), the relative frequency of calms (θ_0) (null wind speeds), maximum recorded wind speed (v_{max}),

[20] The Canary Archipelago is located to the north-west of the African continent, between latitude 27°37′ and 29°25′ north (subtropical position) and longitude 13°20′ and 18°10′ west of Greenwich. The Canary Archipelago is approximately 1000 km from the Spanish mainland coast, and the closest and furthest distances from the African coast are 100 km and 500 km, respectively.

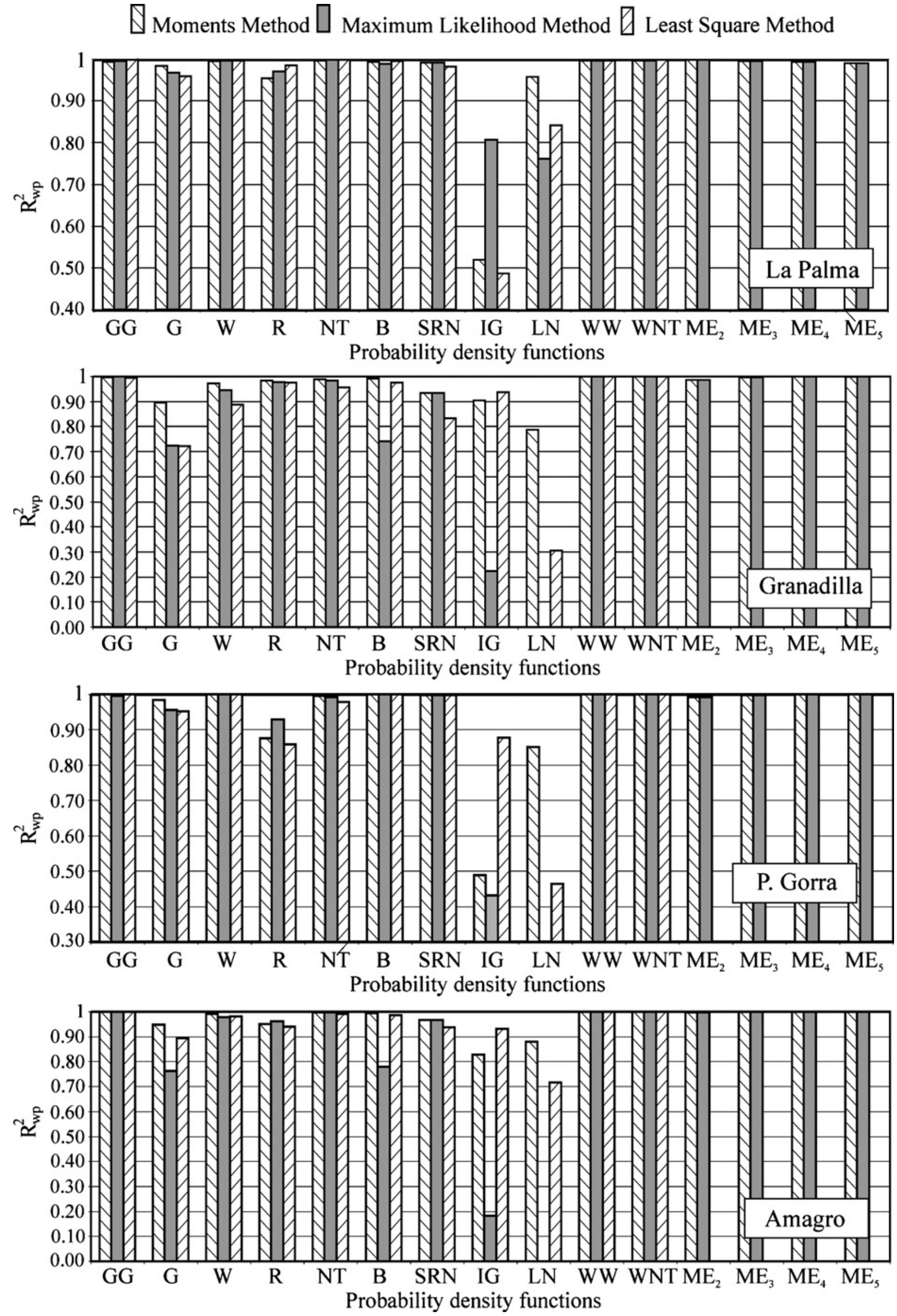

Fig. 2. Coefficients of determination R^2_{wp} obtained with the different non-hybrid PDFs analysed.

coefficient of skewness (Sk), coefficient of kurtosis (Kt) and median (Me). Figs. 3(a), 4(a), 5(a) and 6(a) show the wind speed frequency histograms.

6. Analysis of the results obtained

Tables 2–5 contain the values of the parameters of the different standard distribution laws analysed. Table 6 shows the values of the parameters of the laws generated by applying the principle of maximum entropy. Bearing in mind the work of Siddall [223], in this paper four types of ME_M.pdf[21] have been generated. These PDFs have been generated using 2, 3, 4 and 5 low-order statistical moments with respect to the origin determined numerically from the sample. The Mathcad Software 2001i programme of MathSoft Engineering & Education Inc. [224], was used to find the values of the parameters of the different PDFs analysed.[22]

Fig. 1(a) shows, for the case of La Palma, the values of the coefficients of determination R^2_{ws} obtained with the different PDFs

[21] *M* represents the number of moments used to generate the PDF.

[22] When you solve an equation, by default Mathcad uses an *AutoSelect* procedure to choose an appropriate solving algorithm. In the case of a non-linear system, *AutoSelect* uses the conjugate gradient solver; if that fails to converge, the Levenberg–Marquadt solver; if that too fails, the Quasi–Newton solver. These methods use different algorithms to determine the curvature and direction in which the search is to proceed. Although you can choose a specific method from the popup menu if necessary.

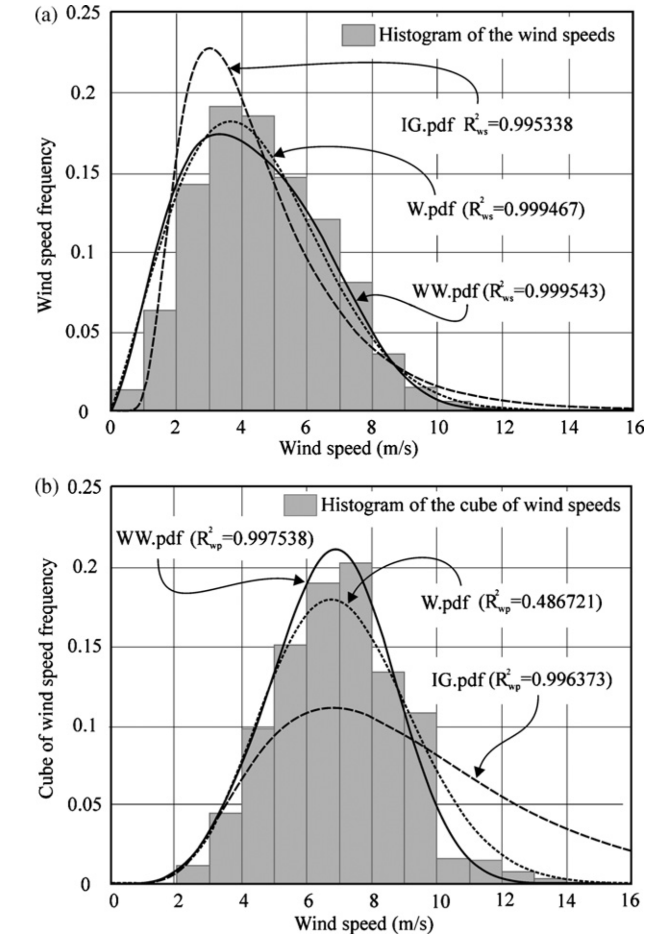

Fig. 3. (a) Relative frequency histogram of wind speed in La Palma on which have been superimposed the Weibull distribution and the distributions which have presented the highest and lowest values of R^2_{ws}; (b) relative frequency histogram of the cube of the wind speed in La Palma on which have been superimposed the Weibull distribution and the distributions which have presented the highest and lowest values of R^2_{ws}.

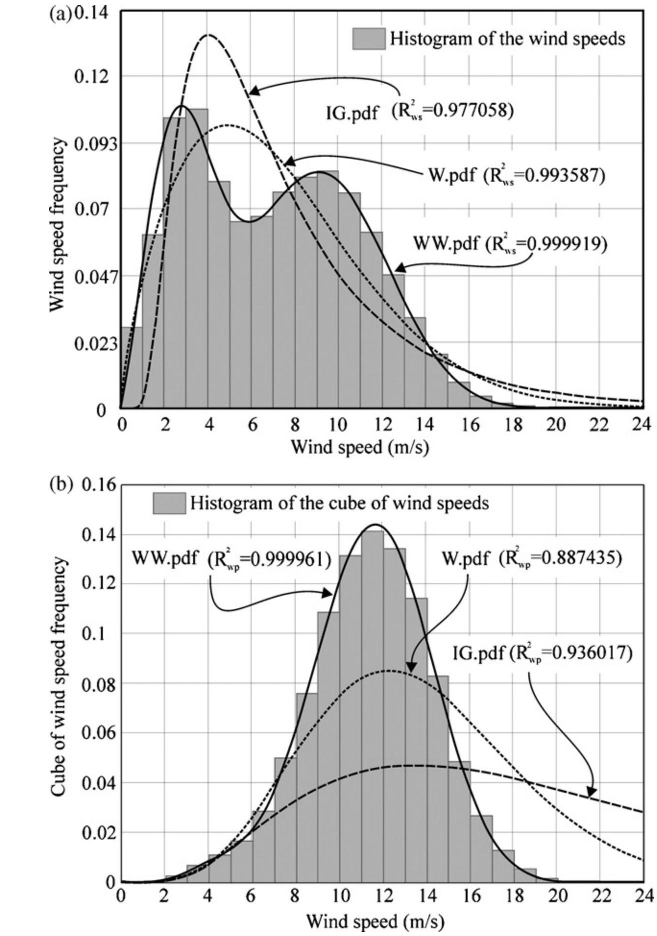

Fig. 4. (a) Relative frequency histogram of wind speed in Granadilla on which have been superimposed the Weibull distribution and the distributions which have presented the highest and lowest values of R^2_{ws}; (b) relative frequency histogram of the cube of the wind speed in Granadilla on which have been superimposed the Weibull distribution and the distributions which have presented the highest and lowest values of R^2_{ws}.

for wind speed.[23] This Fig. 1 also shows the values of R^2_{ws} obtained when the parameters of the PDFs have been estimated using the MM, MLM and LSM. Fig. 1(b)–(d) also represent these coefficients of determination, but in reference to the stations named Granadilla, P. Gorra and Amagro.

Fig. 2(a) shows, for La Palma, the values of R^2_{wp} obtained with the PDFs for wind power density [174,185,202]. Fig. 2(b)–(d) also represent these R^2_{wp}, but in reference to the stations named Granadilla, P. Gorra and Amagro.

It can be seen in Fig. 1 that, independently of the shape of the wind speed histogram and of the type of PDF, it is estimation through LSM which supplies the highest values of R^2_{ws}. However, as can be seen in Fig. 2, this is not always true in the case of wind power densities. In order to achieve high values of R^2_{wp} it is a necessary condition that the PDFs for wind speed have a good fit to the upper tails of the wind speed frequency histograms.[24] This fact can be seen in Figs. 3–6. Figs. 3(a)–6(a) show the wind speed histograms of the stations analysed. The W.pdf and the PDFs which have presented the highest and lowest values of R^2_{ws} have been superimposed on these histograms. Figs. 3(b)–6(c) show the frequency histograms of the cube of the wind speed and the PDFs for wind power density.

It should be pointed out that, with the exception of the PDFs that can be linearized (W.pdf and R.pdf), the estimation of the parameters with LSM requires the use of numerical techniques to resolve Eq. (1). These techniques require a starting point or base point.[25] Good starting values will often allow an iterative technique to converge to a solution much faster than would otherwise be possible. Moreover, inadequate starting values could give rise to Eq. (1) not converging to the optimum solution.[26]

As mentioned in Section 3.1, when the variables are random,[27] the MLM provides a consistent approach to parameter estimation problems and also has desirable mathematical and optimality properties. However, with the exception of certain PDFs that are easily resolved (R.pdf, NR.pdf, LN.pdf and IG.pdf), or which quickly

[23] According to Carta et al. [189], the R^2_{ws} obtained would have been lower if these had been estimated using the probability density functions instead of the cumulative distribution functions.

[24] This is due to the fact that high wind speeds provide much more power than low wind speeds. Then, certain differences between the PDFs for wind speed and the wind speed frequency histograms in the upper tails are transformed into considerable differences between the PDFs for wind power and frequency histograms of the cube of wind speed.

[25] All the iterative procedures require initial values of the parameters to be estimated.

[26] For example, the probability of success with the Newton–Raphson method [217] increases considerable when choosing acceptable initial values of the parameters.

[27] As pointed out in Section 3.3, the wind speed recorded at short intervals of time usually presents dependence.

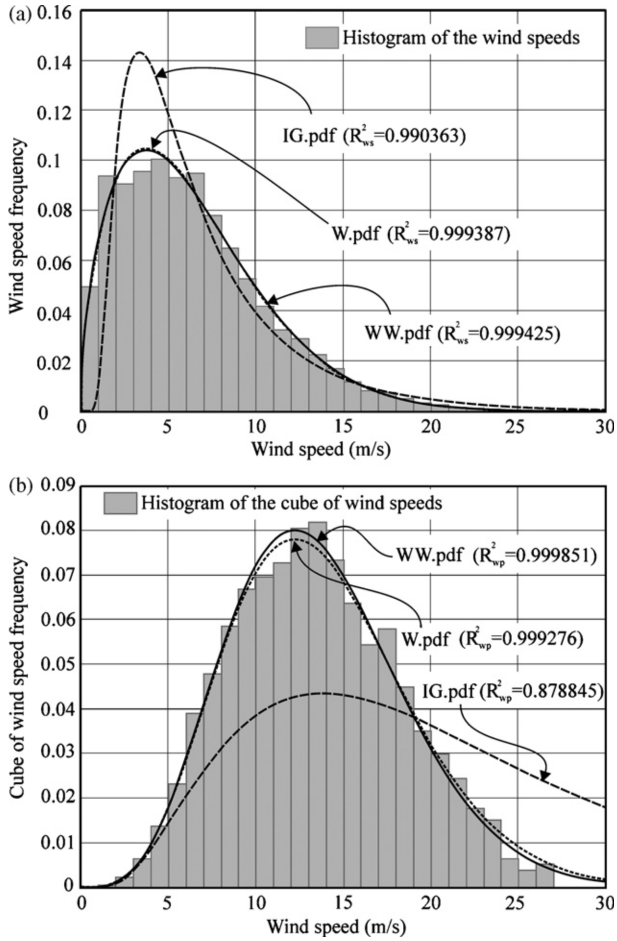

Fig. 5. (a) Relative frequency histogram of wind speed in P. Gorra on which have been superimposed the Weibull distribution and the distributions which have presented the highest and lowest values of R^2_{ws}; (b) relative frequency histogram of the cube of the wind speed in P. Gorra on which have been superimposed the Weibull distribution and the distributions which have presented the highest and lowest values of R^2_{ws}.

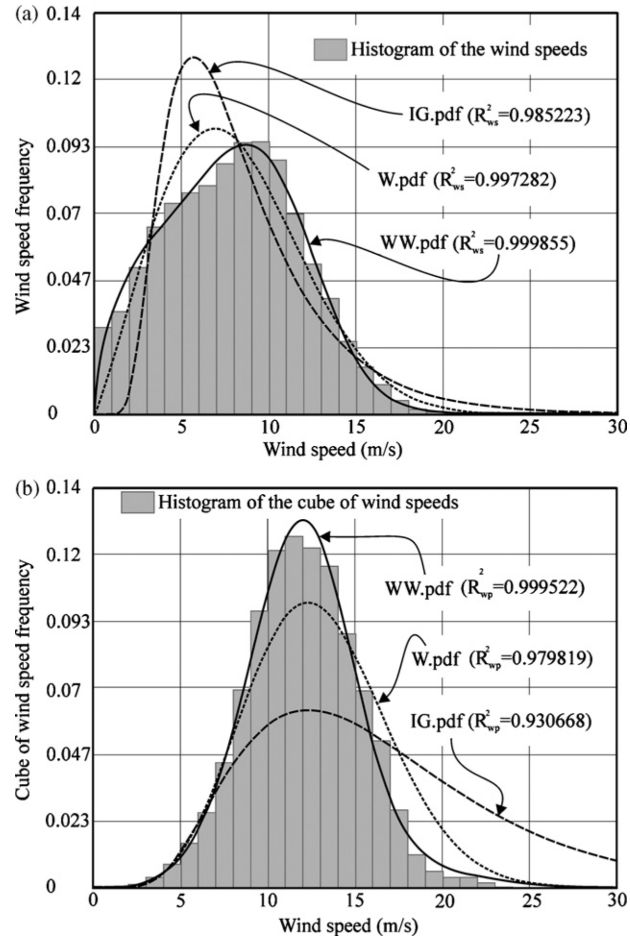

Fig. 6. (a) Relative frequency histogram of wind speed in Amagro on which have been superimposed the Weibull distribution and the distributions which have presented the highest and lowest values of R^2_{ws}; (b) relative frequency histogram of the cube of the wind speed in Amagro on which have been superimposed the Weibull distribution and the distributions which have presented the highest and lowest values of R^2_{ws}.

converge to a solution (W.pdf, G.pdf), non-elementary numerical techniques may be required to estimate the parameters. In this sense, it should be pointed out that the numerical estimation of mixture models is not trivial and that MLM can be sensitive to the initial values of the parameters.

The MM, with the exception of the mixture distributions, the ME_M.pdf and the GG.pdf, do not present difficulties in the estimation of the parameters. Therefore, the parameters obtained using MM can constitute good starting points for estimation of the parameters via MLM or LSM. From the point of view of the values of R^2_{ws} obtained, no rule has been found which advices the use of MM as opposed to MLM or vice versa, in the case of non-random wind speeds.

From the different PDFs analysed the conclusion is reached that the mixture distributions are especially suitable for the representation of bimodal histograms, Fig. 4(a). However in all the unimodal cases considered, Figs. 3(a), 5(a) and 6(a), they have also provided the highest values of R^2_{ws}. The principal drawback, as already mentioned, is that they present greater complexity in the estimation of the parameters, whichever estimation method is used.

For most cases analysed, the IG.pdf and LN.pdf have presented the lowest values of R^2_{ws} and R^2_{wp} (Figs. 1 and 2), independently of the estimation method used. The R.pdf, due to the high rigidity of its shape, could not be fitted to all the wind speed frequency histograms (Figs. 1 and 2). In other words, it is unsuitable for representation of a wide range of wind regimes.

The GG.pdf, due to the fact it has two shape factors (α and η), presents higher flexibility than the W.pdf and, therefore, fits better a larger number of wind speed frequency histograms (Fig. 1). However, the GG.pdf presents the handicap of greater complexity in the estimation of the three parameters on which it depends.

The ME_M.pdf take into account the frequency of null winds [154]. In addition, when the MM has been used, the ME_M.pdf (with $M>2$) has adapted better than most PDFs analysed to the wind speed frequency histograms and, fundamentally, to the frequency histograms of the cube of the wind speed (Fig. 2). Therefore, the ME_M.pdf (with $M>2$) constitute a useful tool for evaluation of the available wind resources at a potential site.

Fig. 7 shows the values of $R^2_{ws,h}$ obtained through the use of hybrid PDFs compared to the values of R^2_{ws} obtained with standard PDFs. The dashed line at 45° has been drawn to observe the relation between both values of the coefficient of determination ($R^2_{ws,h} - R^2_{ws}$). Fig. 8 shows the values of $R^2_{wp,h}$ obtained through the use of hybrid PDFs compared to the values of R^2_{wp} obtained with standard PDFs.

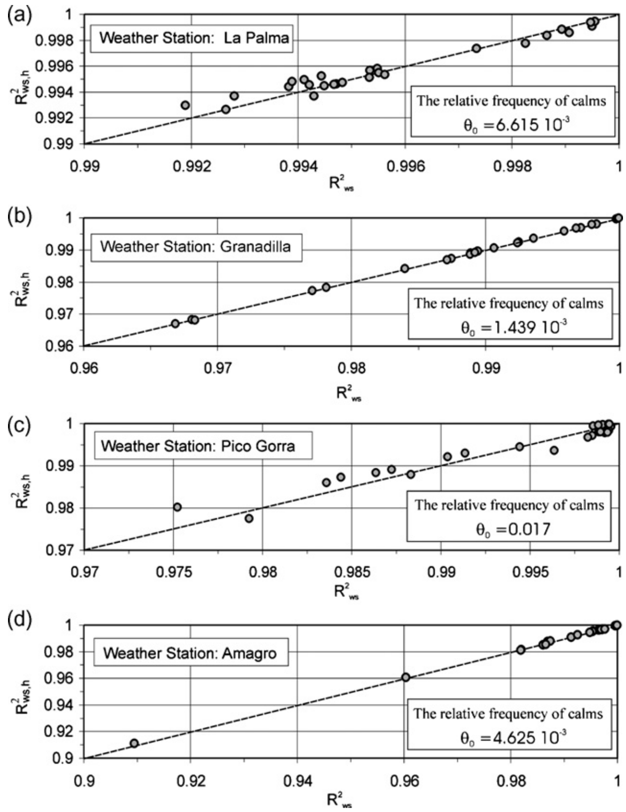

Fig. 7. Values of $R^2_{ws,h}$ obtained through the use of the hybrid distributions against the values of R^2_{ws} obtained through the use of non-hybrid distributions.

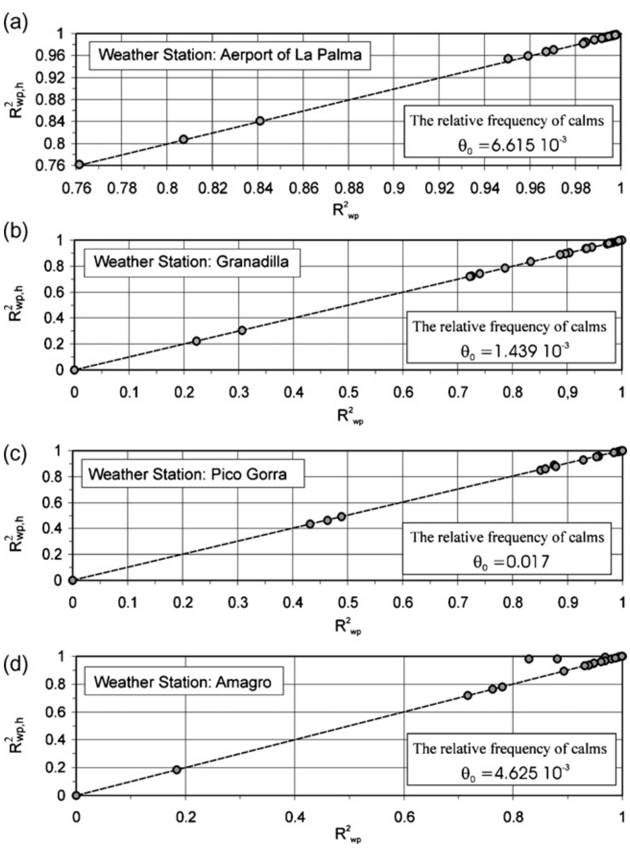

Fig. 8. Values of $R^2_{wp,h}$ obtained through the use of the hybrid distributions against the values of R^2_{wp} obtained through the use of non-hybrid distributions.

As can be seen in Fig. 7, for high values of R^2_{ws} no significant improvement in fit is observed,[28] for any of the stations analysed, when using hybrid PDFs. The stations in La Palma and Pico Gorra have the highest values of null wind speed frequencies (θ_0). At these stations, and for certain values of R^2_{ws}, a slight increase in fit is achieved when using hybrid PDFs. However, in Fig. 8 only a slight increase in fit is observed when using hybrid PDFs at Amagro station and for values of R^2_{wp} between 0.8 and 0.9. Therefore, with respect to the hybrid distributions analysed, and for the values found of θ_0 (<1.7%),[29] the same conclusions are drawn as made by Tuller and Bret [186].

7. Conclusions

In this paper we have seen a review of a variety of PDFs that have been proposed in the scientific literature related to renewable energies to describe wind speed frequency distributions. In addition, new hybrid PDFs have been proposed, such as that defined by Takle and Brown [190], from the standard PDFs that do not take into account the frequencies of null wind speeds.

Details have been given of the analytical and numerical procedures which are required for the estimation of parameters through MM, MLM and LSM. A comparison has also been made in terms of the degree of fit to experimental frequency histograms of wind speed and cube of the wind speed.

The conclusion is reached that the Weibull distribution of two parameters presents a series of advantages with respect to the PDFs analysed. Amongst other advantages, we can mention: (a) its flexibility; (b) the dependence on only two parameters; (c) the simplicity of the estimation of its parameters, independently of the method used; (d) the W.pdf can be expressed in closed form, which simplifies its use; (e) when its parameters are estimated from the sample, it has specific goodness of fit tests [151].

However, the W.pdf cannot represent all the wind regimes encountered in nature such as, for example, those with high percentages of null wind speeds, bimodal distributions, etc. Therefore, its generalised use cannot be justified, and a suitable PDF must be selected for each wind regime in order to minimise errors in the estimation of the energy produced by a WECS [189]. In this sense, the extensive collection of PDFs proposed in this paper constitutes a valuable catalogue.

The mixture distributions of two Weibull distributions or a Weibull distribution and a normal truncated distribution (MWNT.pdf) are particularly suitable for bimodal wind regimes, but they have also provided, for all the unimodal cases considered, the highest values of the coefficient of determination.

The lognormal and inverse Gaussian distributions have presented for all cases analysed the lowest values of R^2_{ws} and R^2_{wp}, independently of the estimation method used. These results contradict the conclusions obtained by other authors [187,199].

The R.pdf is not suitable for representation of a wide range of wind regimes.

The ME_M.pdf (with $M > 2$) constitutes a useful tool for evaluation of wind resources available at a potential site.

The LSM is the method which provides the highest values of R^2_{ws}. However, in order to achieve high values of R^2_{wp} a necessary condition is that the PDFs for wind speed have a good fit to the upper tails of the wind speed frequency histograms.

With the exception of the PDFs that can be linearized (Weibull and Rayleigh), estimation of the parameters through LSM requires

[28] The majority of the points are found over the straight line.

[29] Weather stations with very high values of null wind speeds have not been analysed, since values of θ_0 in the order of 0.49 or 0.52, such as those presented by Merzouk [191], imply winds that are poorly suitable for energy use through a WECS.

the use of numerical techniques and starting points or base points that may complicate the calculations. When the wind speeds are random, the MLM provides a consistent approach to parameter estimation problems and also has desirable mathematical and optimality properties. However, with the exception of certain PDFs which are easily resolved (Rayleigh, square-root normal, lognormal and inverse Gaussian), or which converge rapidly to a solution (Weibull and Gamma), they may require non-elementary numerical techniques for parameter estimation. The MM, with the exception of the mixture distributions, the ME_M.pdf and the GG.pdf, do not present difficulties in parameter estimation. Therefore, the parameters obtained using MM can constitute good starting points for parameter estimation with MLM or LSM and numerical techniques.

From the point of view of the values of R^2_{ws} obtained, no rule has been found that would advise the use of MM as opposed to MLM or vice versa, in the case of non-random wind speeds.

With respect to the hybrid distributions analysed and for the values found of θ_0 ($<1.7\%$), the same conclusions are drawn as made by Tuller and Bret [186]. Namely, there is no indication that hybrid distributions offer advantages over the standard distributions.

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